A new Model for Karst Spring Hydrograph Analysis

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Characterization of Karst Hydrogeology

• Karst environment is very different from other environments in terms of water flow and storage.

• **Direct** methods for characterization: speleology, cave diving, camera recording, borehole logging, remotely-controlled vehicles, tracer experiments, ...
Karst Spring Hydrograph: An Indicator of Karst Aquifers

- Natural experiments of rainfall and karst aquifer responses occur every day.
- **karst spring hydrograph:** the discharge hydrograph appearing at a spring in a karst region where surface flow is almost not possible due to well-developed surface and underground karst landforms (Bonacci, 1993).
- Different hydrographs for different karst types of karst springs.
Karst aquifers do not have a strong capability of storing water. **Hydrograph recession curve**: in a long-lasting period with no precipitation

\[ Q_t = Q_0 e^{-\alpha t} \]

Maillet (1905)
Florida Hydrograph Recession Curve

Hydrograph of Madison Blue Spring

More frequent rainfall events

More complicated karst hydrogeology
Models of Hydrograph Recession Curve

Mangin Model (1975)

\[ \psi(t) \]

\[ \varphi(t) \]

\[ Q_t \]

Fiorillo Model (2011)

Torricelli reservoir (conduit)
Darcy reservoir (matrix)
Poiseuille reservoir (fracture)

Conduit reservoir:
Unsaturated zone flow, \( \Psi(t) \)
Matrix reservoir:
Saturated zone flow: \( \varphi(t) \)
Our Conceptual Model of Flow Dynamics

Two reservoirs: Conduit and matrix (including fracture)

Three stages:
Conduit-flow-dominated, mixed-flow, and matrix-flow-dominated

Explicitly separated conduit and matrix flows in the mixed-flow stage

Conduit-flow-dominated Stage

(a) Stage I: $t_0 \leq t \leq t_1$

Conduit: head decreases from $h_{0,c}$ to $h_{1,c}$

Matrix: head increases from $h_{0,m}$ to $h_{1,m}$

$h_{1,c} = h_{1,m}$

$Q_t^I = Q_0^I - \gamma t$
Mixed-flow Stage with Multiple Conduit Layers

(b) Stage II: \( t_1 \leq t \leq t_2 \)

Conduit: head decreases from \( h_{1,c} \) to \( h_{2,c} \)

\[ h_{2,c} \neq h_{2,m} \]

Matrix: head increases from \( h_{1,m} \) to \( h_{2,m} \)

\[
Q_{t,Li,c}^{II} = Q_{Li,c} - \beta_i (t - t_{Li,c})
\]

\[
Q_{t,m}^{II} = Q_{1,m} e^{-\alpha_1(t-t_1)}
\]

Matrix-flow-dominated Stage

(c) Stage III: \( t_2 \leq t \)

Conduit: head remains at \( h_{2,c} \)

Matrix: head continues decreasing

\[
Q_{t}^{III} = Q_{2,m} e^{-\alpha_2(t-t_2)}
\]
**Idealized Hydrograph Separation**

\[ Q^I_t = Q^I_0 - \gamma t \]

Spring discharge \((Q)\) is linear with time \((t)\).

**Conduit-flow-dominated stage**  
(Stage I)

- \( V^{II}_{L1,c} \): conduit flow from layer 1
- \( V^{II}_{L2,c} \): conduit flow from layer 2
- \( V^{II}_m \): matrix flow

**Matrix-dominated-flow stage**  
(Stage III)

\[ Q^{III}_t = Q_{2,m} e^{-\alpha_2(t-t_2)} \]

Logarithm of spring discharge \((\log Q)\) is linear with time \((t)\).
Real-World Application and Model Comparison

- Madison Blue Spring located in SRWMD
- Two periods are selected for model application and evaluation
  - Recession period 1: small conduit flow
  - Recession period 2: large conduit flow
When conduit flow is small, the three models fit the data almost equally well.

\[
\text{misfit} = \sum_{i=1}^{n} |r_i|
\]

Our model: 0.433 m³/s
Mangin model: 0.462 m³/s
Fiorillo model: 0.449 m³/s
When conduit flow is large, our model outperforms the other two models. The misfit is given by:

\[
\text{misfit} = \sum_{i=1}^{n} |r_i|
\]

Our model: 1.895 m$^3$/s
Mangin model: 4.286 m$^3$/s
Fiorillo model: 2.747 m$^3$/s
Our model

Mangin model: not considering three stages of spring discharge.
Fiorillo model: not separating conduit flow and matrix flow in the mixed-flow stage.

Why is our model better?

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Mangin model (1975)

Fiorillo model (2011)
Effective Porosity of Matrix and Conduit

(b) Stage II: $t_1 \leq t \leq t_2$

$$n_m = \frac{V^{II}_m}{(h_{1,m} - h_{2,m}) A_c}$$

$$V^{II}_m = \int_{t_1}^{t_2} Q^{II}_{t,m} dt = \int_{t_1}^{t_2} Q_{1,m} e^{-\alpha_1(t-t_1)} dt = \frac{Q_{1,m} - Q_{2,m}}{\alpha_1}$$
Effective Porosity of Matrix and Conduit

(b) Stage II: $t_1 \leq t \leq t_2$

Conduit Layer $L_i$

$$n_{L_i,c} = \frac{V_{L_i,c}^{II}}{(h_{Lis,c} - h_{Lie,c}) A_c}$$

$$V_{L_i,c}^{II} = \int_{t_{Lis,c}}^{t_{Lie,c}} Q_{t,L_i,c}^{II} dt = \frac{Q_{Lis,c}^{2} - Q_{Lie,c}^{2}}{2 \beta_i}$$

$$h_{Lie,c} = (h_{Lis,c} - h_{2,c}) \left( \frac{Q_{t,L_i,c}^{II}}{Q_{t,L_i+1,c}^{II}} \right)^2 + h_{2,c}$$
## Results

<table>
<thead>
<tr>
<th>Starting Date</th>
<th>Head range (m)</th>
<th>Released water volume (m$^3$)</th>
<th>Effective porosity (%)</th>
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</thead>
<tbody>
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<td>Matrix</td>
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</tbody>
</table>

![Graph](image1.png)

(a) Sept. 20, 2014

![Graph](image2.png)

(b) Sept. 13, 2016
## Results

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- Two boreholes, W-15537 and W-15515, were completed in 1984.
  - W-15537: 5% ~ 16%
  - W-15515: 1% ~ 26 %
- How do I know that the estimated conduit porosity is not wrong?
- Is it reasonable that the matrix flow is significantly larger than the conduit flow during the mixed-flow stage?
Conclusions

• A new model is developed for simulating the recession periods of karst spring hydrograph.

• The application of the new model to the data of the Madison Blue Spring indicates that the new model outperforms the Mangin model and Fiorillo model.

• The new model enables the estimation of effective porosity of matrix and conduit during the mixed-flow stage.

• Limitations:
  ➢ The karst spring hydrograph must have the matrix-flow-dominated stage so that the conduit flow and matrix flow in the mixed-flow stage can be separated.
  ➢ The model requires several parameters that cannot be directly measured, such as the area of springshed and the hydraulic head of the conduit reservoir at the beginning of the matrix-flow-dominated stage.